

Straight Lines

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of A.
- (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
- (c) (A) is true but (R) is false.
- (d) (A) is false but (R) is true.

1. Assertion (A): The angle between the line $x+2y-3=0$ and $3x+y+1=0$ is $\tan^{-1}(1)$.

Reason (R): Angle between two lines is

$$\text{given by } \tan \theta = \pm \left(\frac{m_2 - m_1}{1 + m_1 m_2} \right)$$

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Let m_1 and m_2 be the slopes of the straight lines $x+2y-3=0$ and $3x+y+1=0$.

$$\text{Then, } m_1 = -\frac{1}{2} \text{ and } m_2 = -3$$

Let θ be the angle between the given lines.

$$\begin{aligned} \text{Then, } \tan \theta &= \pm \left(\frac{m_2 - m_1}{1 + m_1 m_2} \right) \\ &= \pm \left(\frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}} \right) = \pm 1 \end{aligned}$$

$$\Rightarrow \theta = \tan^{-1}(1)$$

2. Assertion (A): A slope of line $3x-4y+10=0$

is $\frac{3}{4}$.

Reason (R): x-intercepts and y-intercepts of $3x-4y+10=0$ respectively are

$$y = \frac{3}{4}x + \frac{5}{2}$$

Ans. (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Explanation: Given equation $3x-4y+10=0$ can be written as

$$y = \frac{3}{4}x + \frac{5}{2}$$

Comparing eq. (1) with $y = mx + c$, we have slope
of the given line as $m = \frac{3}{4}$.

Equation $3x-4y+10=0$ can be written as

$$3x - 4y = -10 \text{ or } \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1 \quad \dots(ii)$$

Comparing eq. (ii) with $\frac{x}{a} + \frac{y}{b} = 1$, we have

x-intercept as $a = -\frac{10}{3}$ and y-intercept as

$$b = \frac{5}{2}.$$

3. Assertion (A): If $x \cos \theta + y \sin \theta = 2$ is
perpendicular to the line $x - y = 3$
then one of the values of

$$\theta \text{ is } \frac{\pi}{4}.$$

Reason (R): If two lines $y = m_1x + c_1$ and
 $y = m_2x + c_2$ are perpendicular
then $m_1 = m_2$

Ans. (c) (A) is true but (R) is false.

Explanation: Since, slope of line $x \cos \theta + y \sin \theta = 2$ is $\cot \theta$ and slope of line $x - y = 3$ is 1.
Also, these lines are perpendicular to each other.

$$\therefore (-\cot \theta)(1) = -1$$

$$\Rightarrow \cot \theta = 1$$

$$\Rightarrow \theta = \cot^{-1} \left(\cot \frac{\pi}{4} \right)$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Condition of perpendicularity of two lines is $m_1 \cdot m_2 = -1$ and not $m_1 = m_2$

4. Assertion (A): The slope of the line $x + 7y = 0$

is $\frac{1}{5}$ and y -intercept is 0.

Reason (R): The slope of the line $6x + 3y - 5 = 0$

is -2 and y - intercept is $\frac{5}{2}$.

Ans. (d) (A) is false but (R) is true.

Explanation: Given equation is $x + 7y = 0$

$$\Rightarrow y = \frac{-x}{7} + 0$$

On comparing with $y = mx + c$, we get

$$\text{Slope } (m) = \frac{-1}{7}, y - \text{intercept} = 0$$

Given equation is $6x + 3y - 5 = 0$

$$\Rightarrow y = -2x + \frac{5}{3}$$

On comparing with $y = mx + c$, we get

$$\text{Slope } (m) = -2, y - \text{intercept} = \frac{5}{3}$$

5. Assertion (A): Slope of x -axis is zero and slope of y -axis is not defined.

Reason (R): Slope of x -axis is defined and slope of y -axis is zero.

Ans. (c) (A) is true but (R) is false.

Explanation: slope of x-axis is zero and hence defined but slope of y-axis is not defined.

6. Assertion (A): The distance between the line

$$4x + 3y = 11 \text{ and } 8x + 6y = 15$$

is $\frac{7}{10}$.

Reason (R): The distance between the line

$ax + by = c_1$ and $ax + by = c_2$ is

$$\text{given by } \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|.$$

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Given lines are

$$4x + 3y = 11$$

and $2(4x + 3y) = 15$

i.e., $4x + 3y = \frac{15}{2}$

Distance between them is

$$\begin{aligned} &= \left| \frac{11 - \frac{15}{2}}{\sqrt{16 + 9}} \right| \\ &= \left| \frac{7}{10} \right| = \frac{7}{10} \end{aligned}$$

